OSCILLATIONS

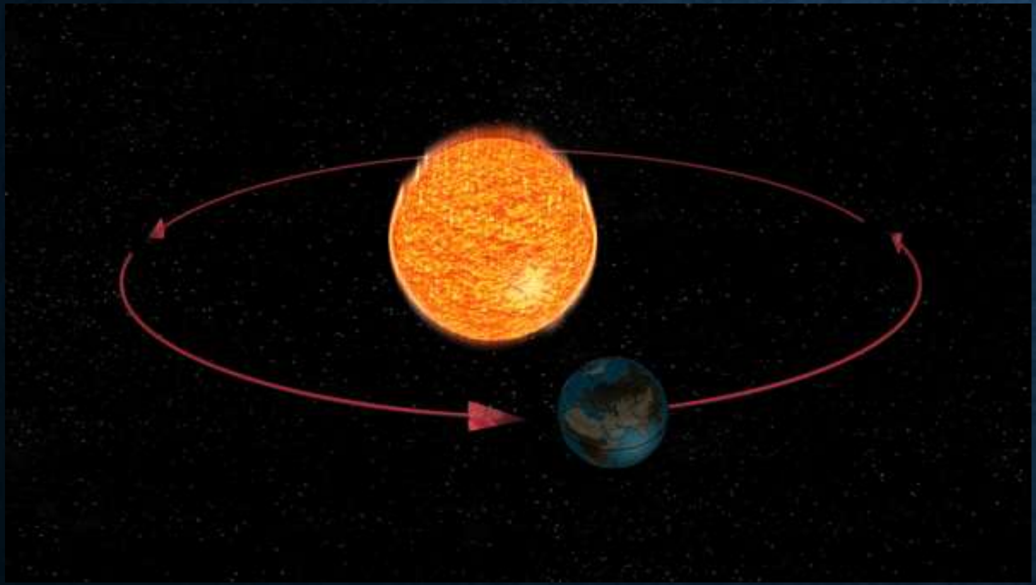
SIMPLE HARMONIC MOTION

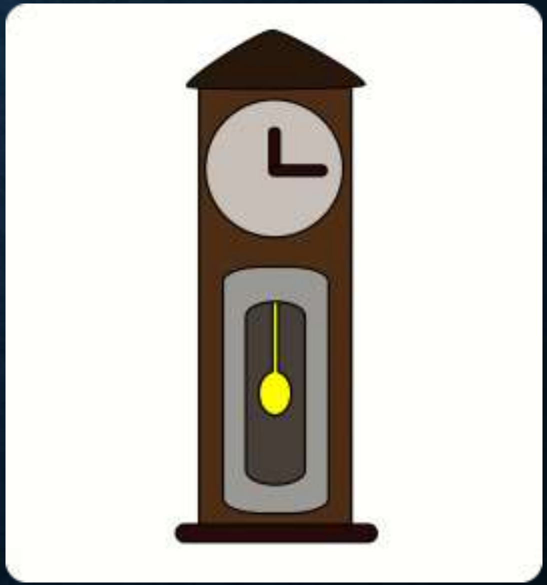
CONTENT

- *Periodic motion*
- *Oscillatory motion*
- *Simple harmonic motion (SHM)*
- *SHM and Circular motion*
- *Velocity and acceleration in SHM*
- *Energy in SHM*
- *Some systems executing SHM*
    1. *Oscillations due to spring*
    2. *Simple Pendulum*
- *Damped Oscillations*
- *Forced and Resonance oscillations*

PERIODIC MOTION

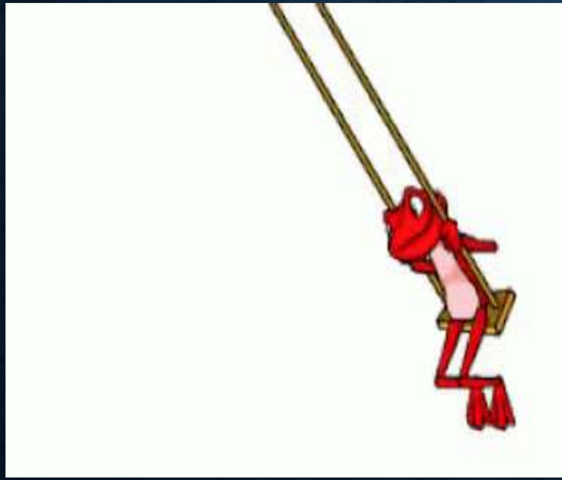
- *A motion which repeats itself over and over again after a regular interval of time is called a periodic motion.*
- *Examples: Revolution of earth around sun, motion of swing, pendulum of clock etc.*

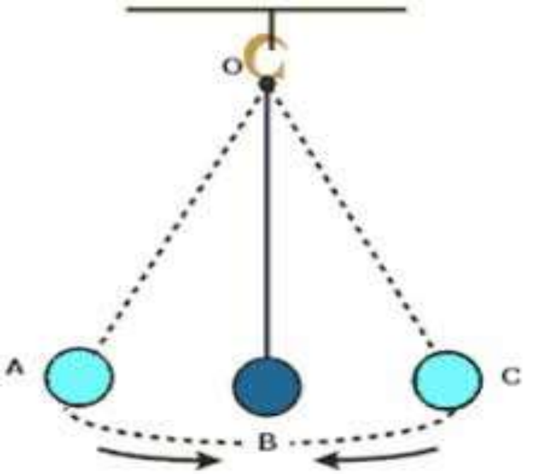




OSCILLATORY OR VIBRATORY MOTION

- *It is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point in a definite interval of time.*
- *Every oscillatory motion is periodic motion but every periodic motion need not to be oscillatory.*
- *Examples : motion of swing, motion of simple pendulum etc.*

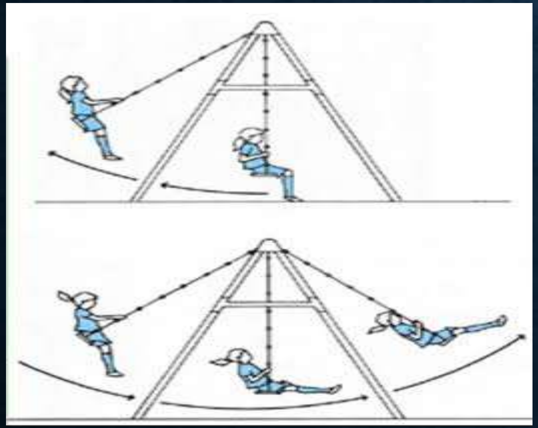




DIFFERENCE B/W OSCILLATIONS AND VIBRATIONS :

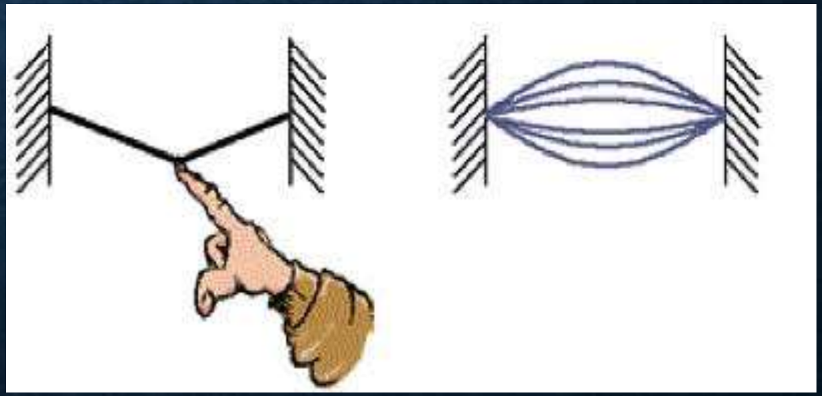
Oscillations

- ***When frequency is small,we call it oscillation.***
- ***e.g Oscillations of a branch of a tree, motion of swing etc.***



Vibrations

- ***When frequency is high,we call it vibration.***
- ***e.g vibration of a string of a musical instrument , motion of wings of fly etc.***



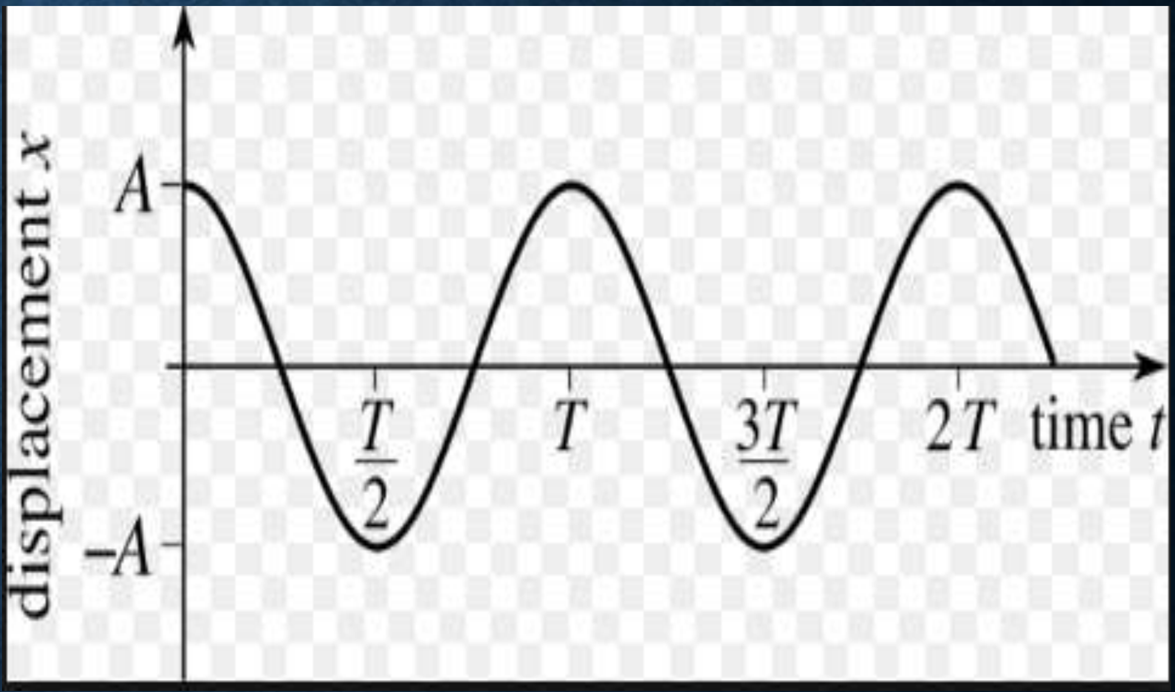
SOME IMPORTANT TERMS:

- ***Amplitude:*** The amplitude of particle executing SHM is its maximum displacement on either side of the mean position.
- ***Time Period:*** Time period of a particle executing SHM is the time taken to complete one cycle and is denoted by T. Its S.I. unit is second.
- ***Frequency:*** The frequency of a particle executing SHM is equal to the number of oscillations completed in one second. It is reciprocal of time period.
- ***Phase:*** The phase of particle executing SHM at any instant is its state as regard to its position and direction of motion at that instant. it is measured as argument (angle) of sine in the equation of SHM. It is represented by Φ.

SIMPLE HARMONIC MOTION:

- It is a specific type of oscillatory motion, in which

- **Particle moves in one dimension.**
- **Particle moves to and fro about a fixed mean position (where Fnet = 0).**
- **Net force on the particle is always directed towards means position.**
- **Magnitude of net force is always proportional to the displacement of particle from the mean position at that instant. So, Fnet = –kx where, k is known as force constant**



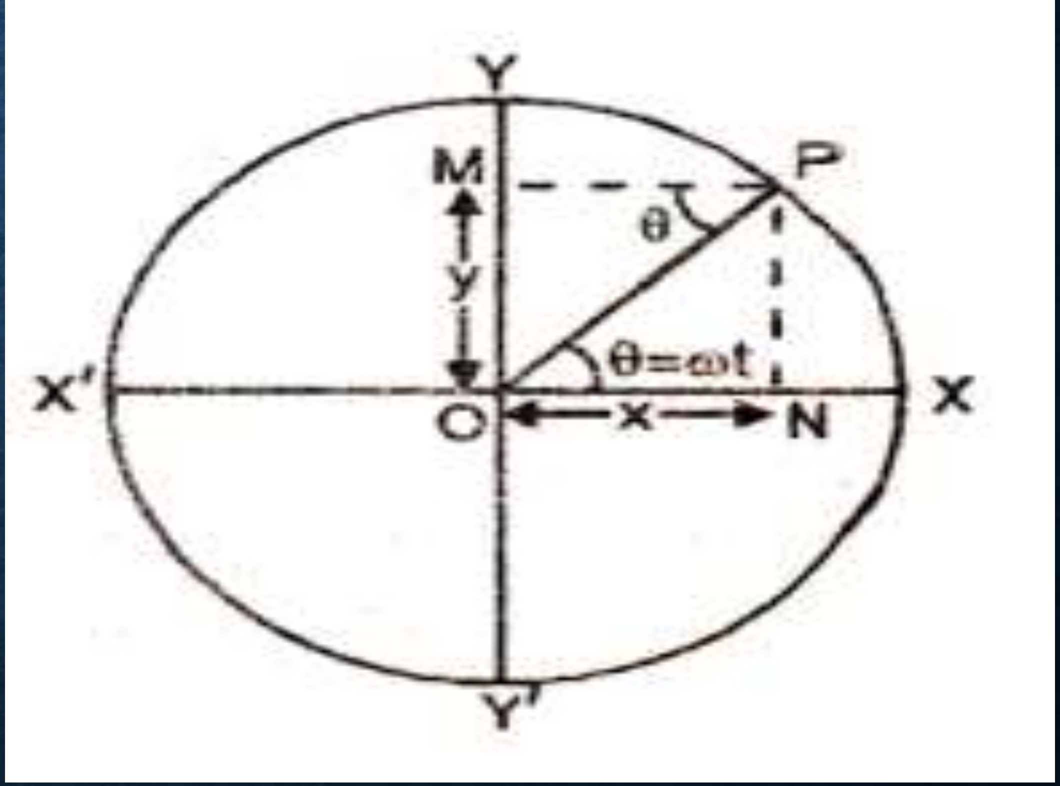
SIMPLE HARMONIC MOTION IS THE PROJECTION OF UNIFORM CIRCULAR MOTION ON ANY DIAMETER

- From the figure

$$\sin \omega t = \frac{OM}{OP}$$

Therefore, OM = OP sin ωt

So y = r sin ωt



VELOCITY IN SHM

Displacement is given by

$$y = A \sin(\omega t+\phi) \quad \text{..........(1)}$$

On differentiating equation (1) with respect to time, We get the velocity of the particle executing SHM.

$$\frac{dy}{dt} = \frac{d}{dt}(A \sin(\omega t + \phi))$$

$$v = A\,\omega \cos(\omega t + \phi)$$

$$v = \omega \sqrt{(A^2 - y^2)} \quad \text{..........(2)}$$

Note:

1. Velocity is minimum at extremes because the particles is at rest. i.e.,
v = 0 at extreme position.
2. Velocity has maximum magnitude at mean position.

ACCELERATION IN SHM

On differentiating equation (2) with respect to time

$$\frac{dy}{dt} = \frac{d}{dt}(A\omega\cos(\omega t + \phi))$$

$$a = -A\omega^2 \sin(\omega t + \phi)$$

$$a = -\omega^2 y$$

Note:

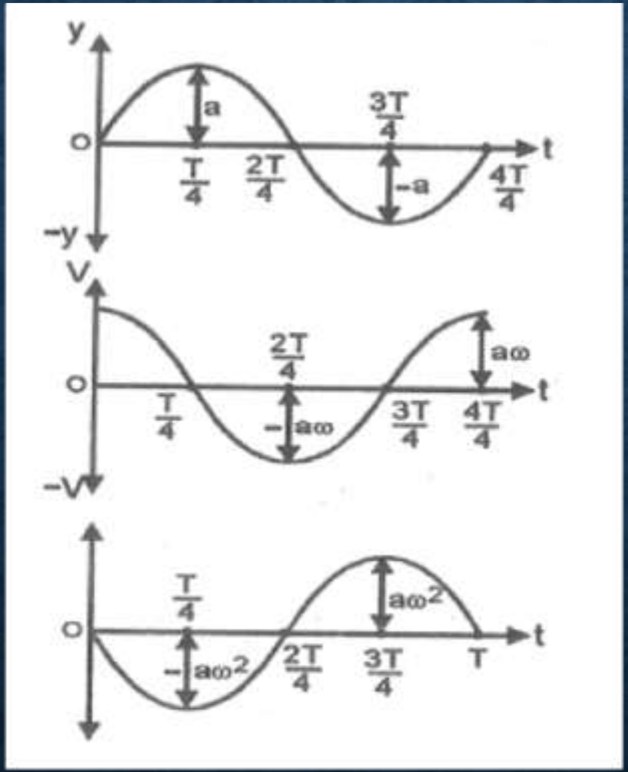
1. Acceleration is always directed towards mean position.
2. The magnitude of acceleration is minimum ($|a|_{min} = 0$) at mean position and maximum at extremes.

GRAPHICAL REPRESENTATION OF DISPLACEMENT, VELOCITY AND ACCELERATION:

Displacement with time

Velocity with time

Acceleration with time



VARIOUS ENERGIES OF PARTICLE EXECUTING SHM:

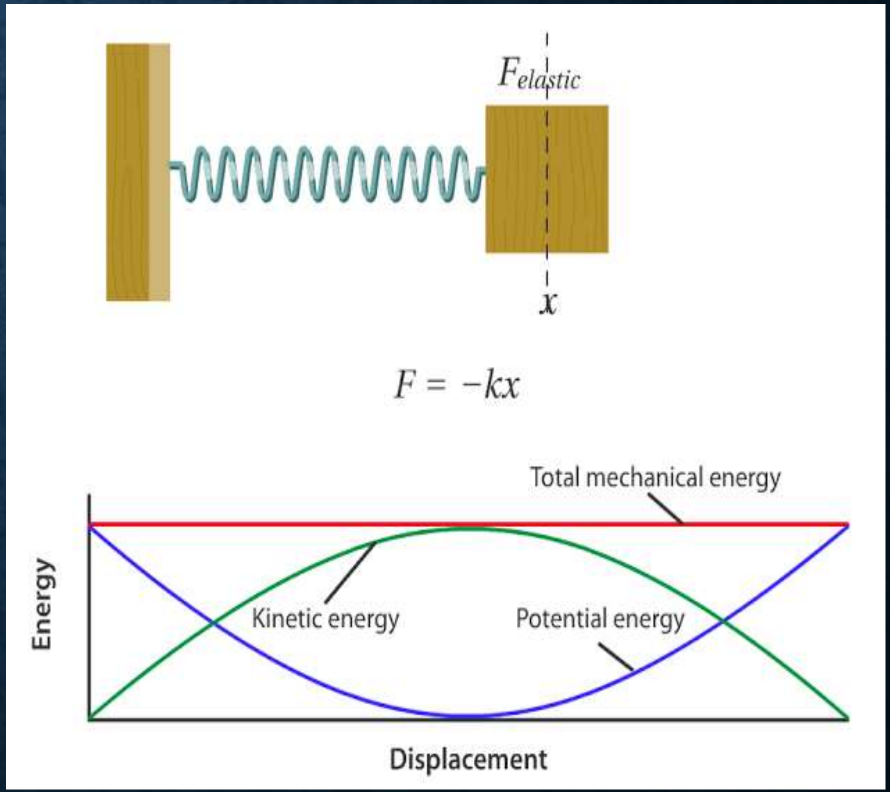
- Kinetic energy K $= \frac{1}{2} m v^2$

$$= \frac{1}{2} m\omega^2 A^2 sin^2(\omega t + \varphi)$$

$$= \frac{1}{2} kA^2 sin^2(\omega t + \varphi)$$

Potential energy U $= \frac{1}{2} kx^2$

$$= \frac{1}{2} kA^2 cos^2(\omega t + \varphi)$$



TOTAL ENERGY IN SHM:

- The total energy is the sum of kinetic energy and potential energy.

Total energy E = K + U

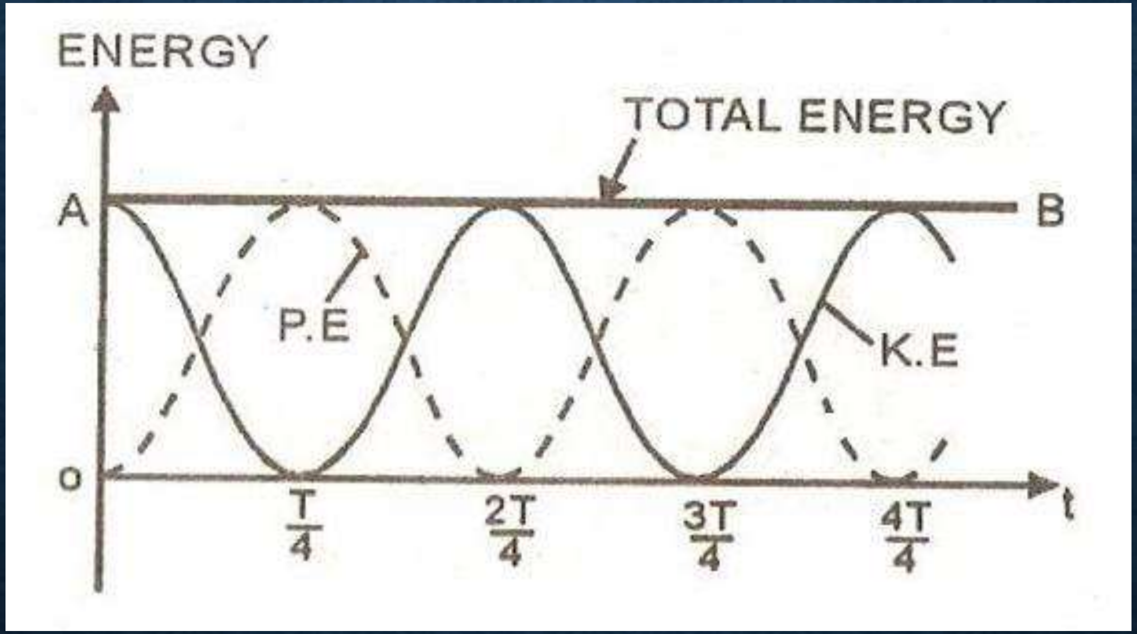
$$= \frac{1}{2}kA^2 sin^2(\omega t + \varphi) + \frac{1}{2}kA^2 cos^2(\omega t + \varphi)$$

$$= \frac{1}{2}kA^2 \{sin^2(\omega t + \varphi) + cos^2(\omega t + \varphi)\}$$

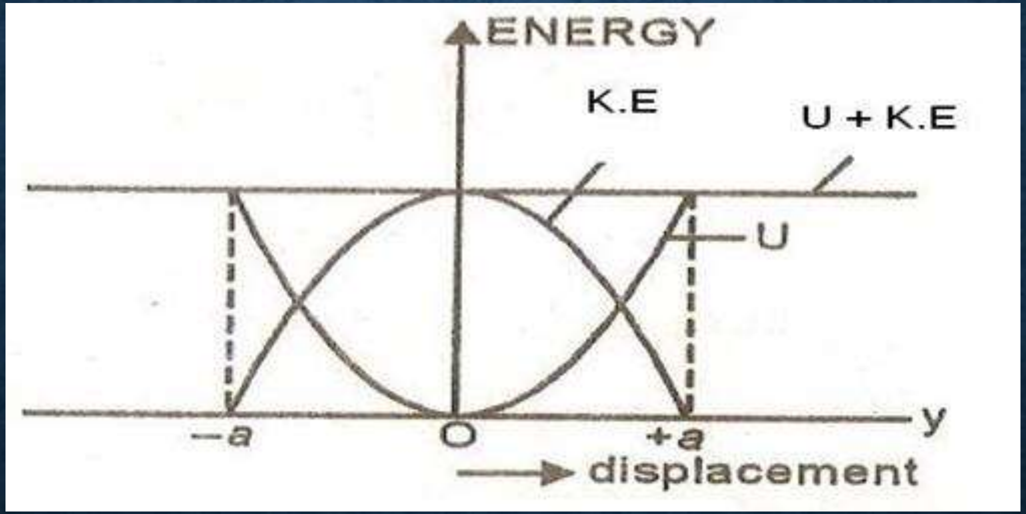
$$E = \frac{1}{2}kA^2$$

Note: Total enegy is independent of time and displacement.

GRAPHICAL REPRESENTATION OF ENERGY W.R.T TIME



GRAPHICAL REPRESENTATION OF ENERGY W.R.T DISPLACEMENT:



OSCILLATIONS IN HORIZONTAL SPRING:

- Force in loaded spring is , F = - kx

$$m\frac{d}{dt}\left(\frac{dy}{dt}\right) = -kx$$

$$\frac{d}{dt}\left(\frac{dy}{dt}\right) + \left(\frac{k}{m}\right) x = 0$$

comparing this equation with general equation of SHM , we get

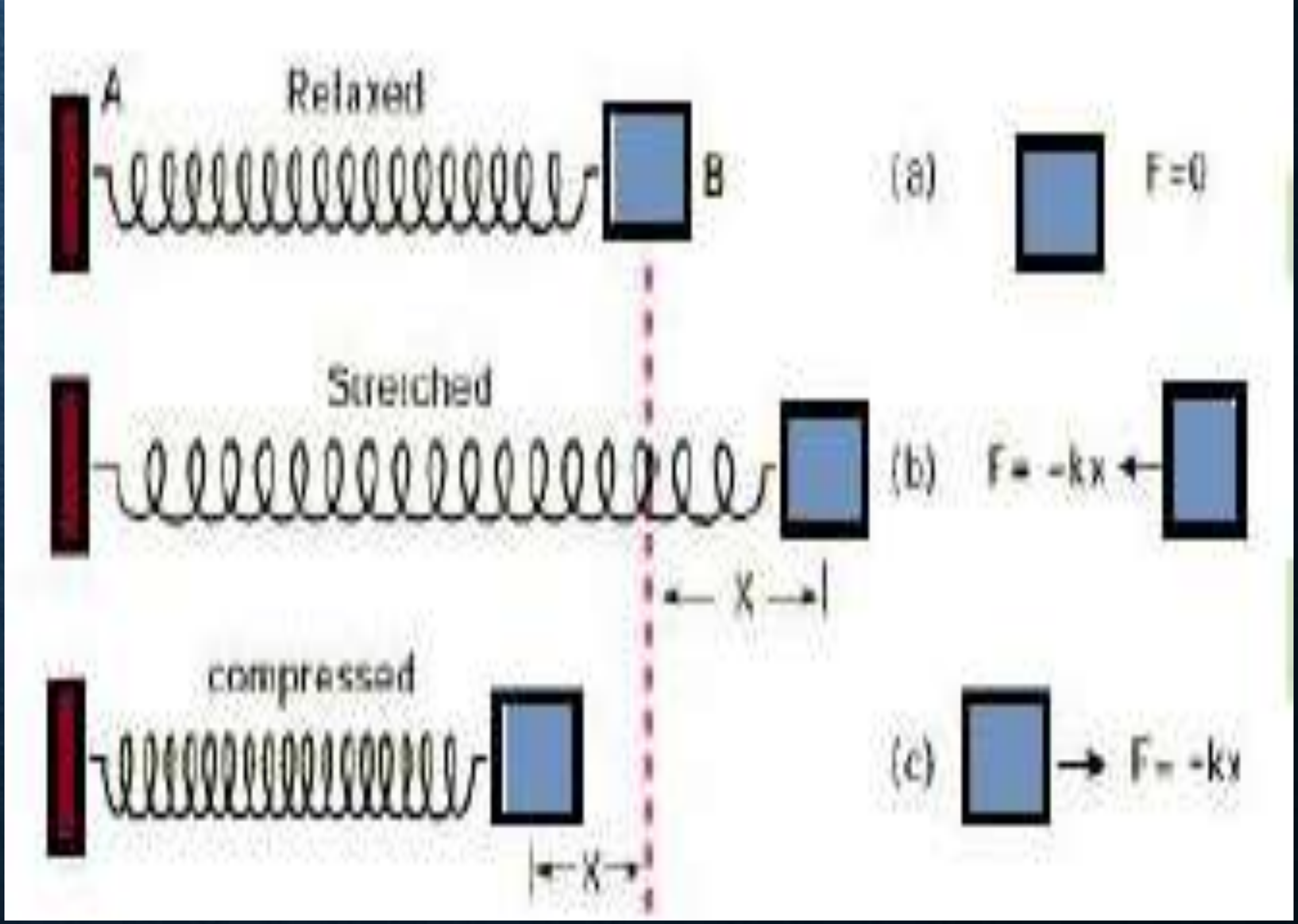
$$\omega = \sqrt{\frac{k}{m}}$$

Time period of SHM is

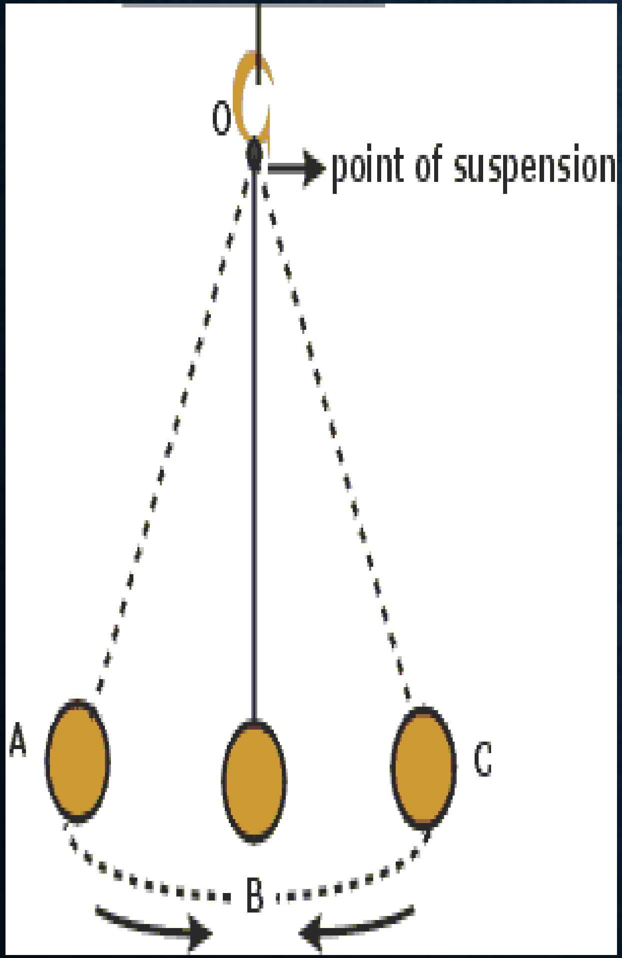
$$T = \frac{2\pi}{\omega}$$

So, time period of loaded spring is

$$T = 2\pi\sqrt{\frac{m}{k}}$$



SIMPLE PENDULUM:



- A small bob of mass m tied to an inextensible mass less string of length L is called d a simple pendulum.

Restoring torque produced is

$$\tau = - L \sin\theta$$

Acceleration produced

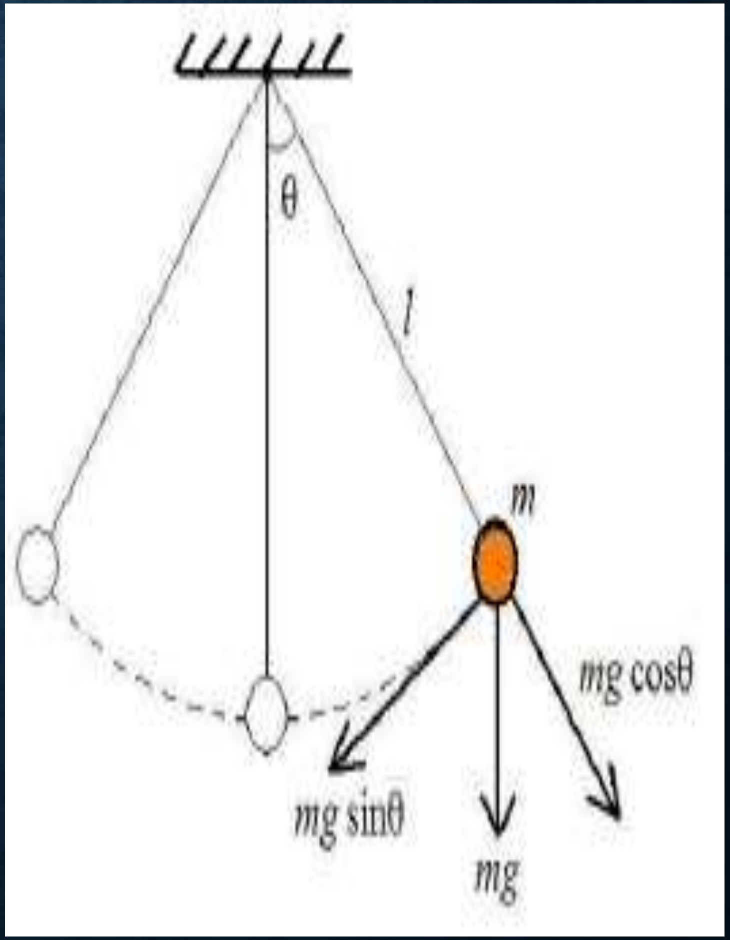
$$a = -\frac{mgL}{I}\theta$$

Angular frequency

$$\omega = \sqrt{(g/L)}$$

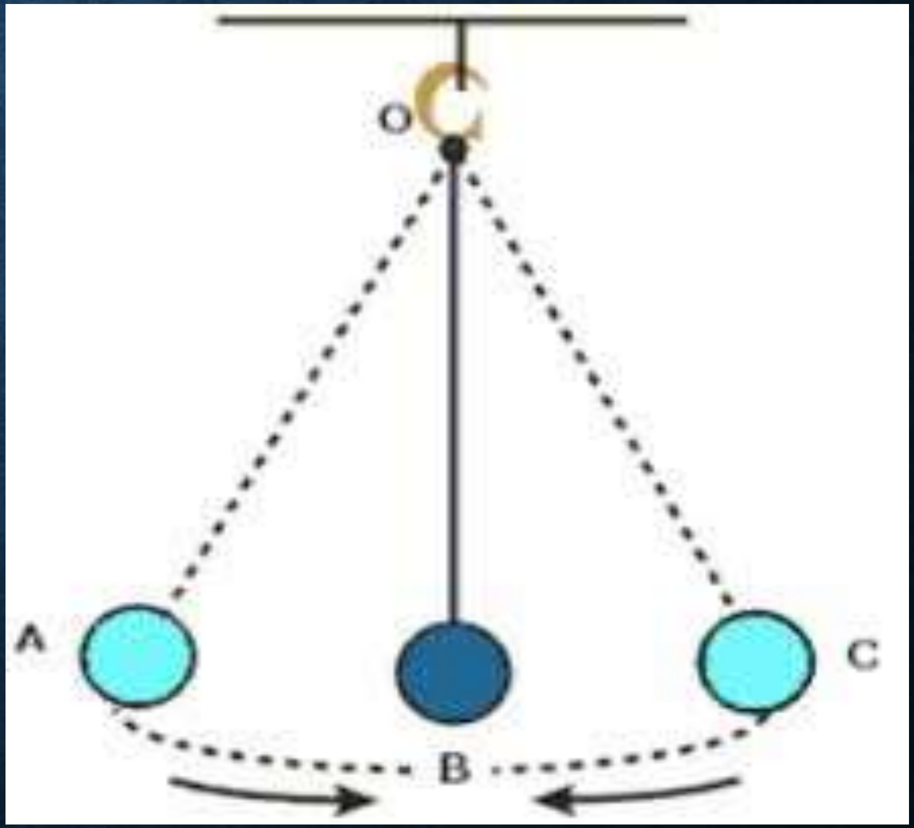
So, Timeperiod of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$



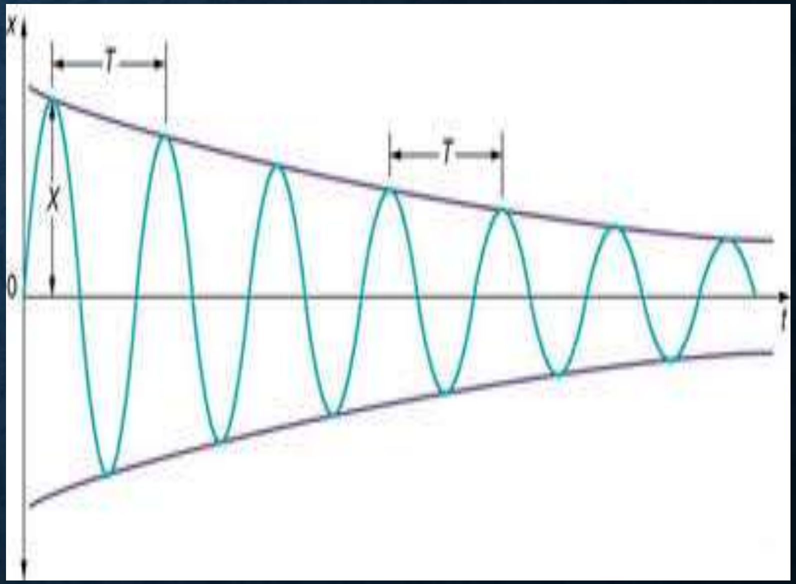
FREE OSCILLATIONS:

- When a system is displaced from its equilibrium position and released, it oscillates with its natural frequency and oscillations are called free oscillations.
- All free oscillations eventually die out because of ever present damping forces.



DAMPED OSCILLATION

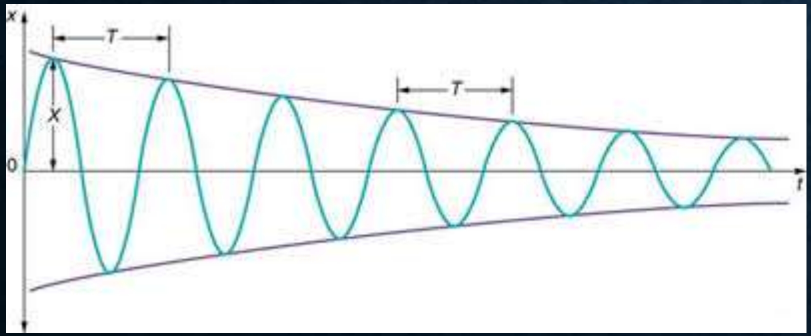
- *The oscillation of a body whose amplitude goes on decreasing with time is defined as damped oscillation.*
- *In this oscillation the amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force etc.*
- *Due to decrease in amplitude the energy of the oscillator also goes on decreasing exponentially*.

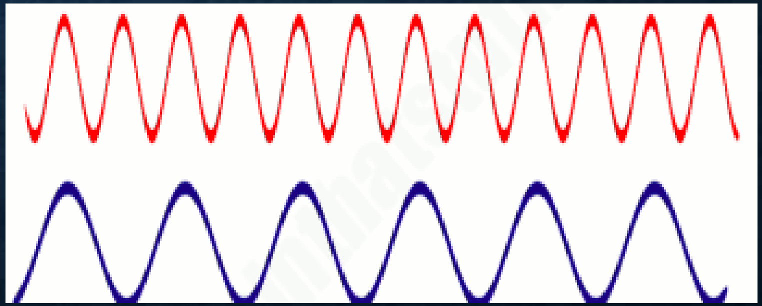


DAMPED OSCILLATION V/S UNDAMPED OSCILLATION

- In damped oscillations amplitude decreases with time.
- Such type of oscillations does not continue for longer time.
- Damped oscillators will die out eventually.

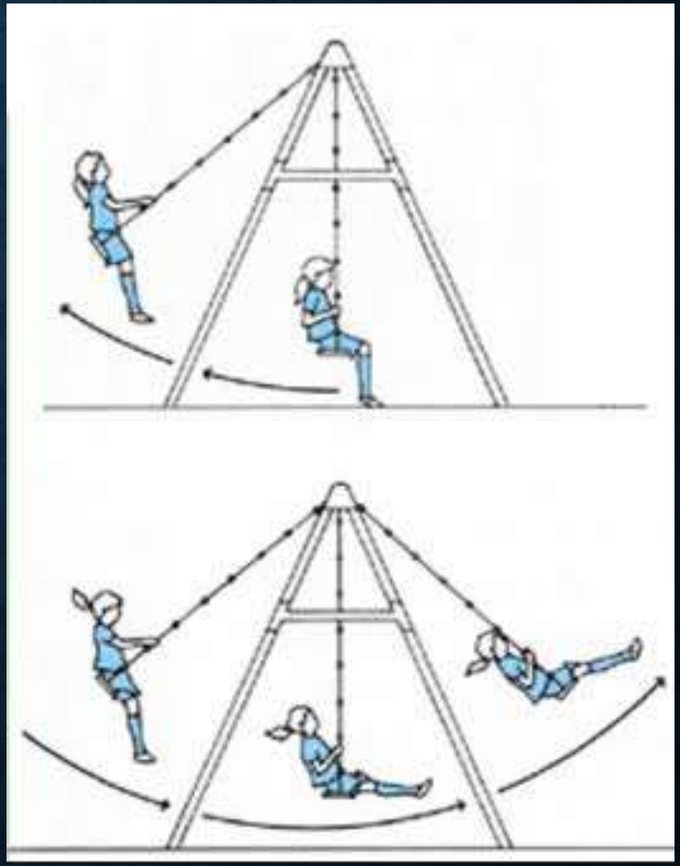
- In undamped oscillations amplitude remains constant.
- Either there is no powerloss or there is a provision to compensate for the power losses.
- Undamped oscillators will oscillate indefinitely.





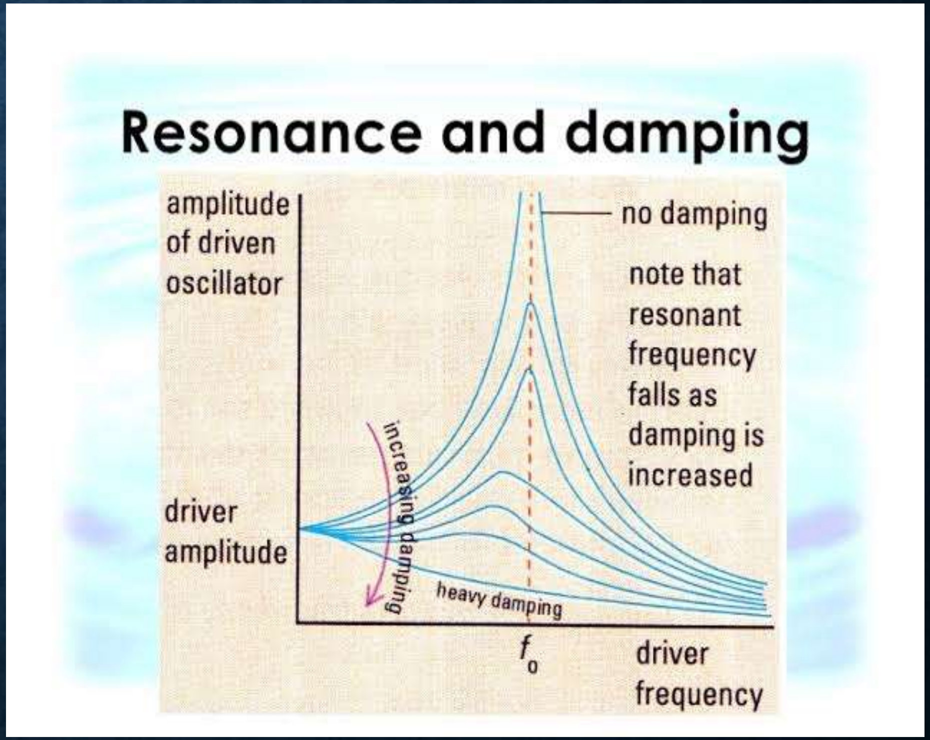
FORCED OSCILLATION :

- *The oscillation in which a body oscillates under the influence of an external periodic force are known as forced oscillation.*
- *System oscillates with frequency of external agency not with natural frequency.*
- *Energy of the body is maintained constant by external periodic force.*
- *These are also known as derived oscillations.*
- *Example- A child in a garden swing periodically presses his feet against the ground to maintain the oscillations.*



RESONANCE :

- ***When the frequency of external force is equal to the natural frequency of the oscillator, then this state is known as the state of resonance. And this frequency is known as resonant frequency.***
- ***In the ideal case of zero damping, amplitude tends to infinity.***
- ***Skill in swinging to greater heights lies in the synchronization of the rhythm of pushing against the ground with the natural frequency of the swing.***



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